final manuscript

– Submission to Logique & Analyse –

PROOF-THEORETIC SEMANTICS AND HYPERINTENSIONALITY

Ivo Pezlar

Abstract

In his recent book *The Impossible: An Essay on Hyperintensionality* (2014) Jago states that proof-theoretic semantics (PTS) does not easily deliver hyperintensional contents. I argue against this claim and show that, on the contrary, hyperintensionality is one of the basic features of PTS approaches.

Keywords: Proof-theoretic semantics · Hyperintensionality · Semantic fine-grainedness · Granularity of meaning · Inferentialism · Hyperintensional logic

1 Introduction and Motivation

Jago's argument against hyperintensionality of proof-theoretic semantics (PTS) can be found in chapter 3 of his recent book *The Impossible: An Essay on Hyperintensionality* (2014). It is subsumed under his criticism of conceptual role semantics (CRS), which he views as a generalization of PTS (Jago 2014, p. 67), and its main conclusion is that (Jago 2014, p. 68):

> Claim (**A**): Proof-theoretic semantics does not easily deliver hyperintensional contents.

which is taken to be supported by the following claim (Jago 2014, p. 67):

> Claim (**B**): In proof-theoretic semantics every logically equivalent proposition has the same content.

In this paper I want to argue against these two claims. More specifically, I will show that, contrary to the above claims, hyperintensionality is not difficult to obtain in the settings of proof-theoretic semantics (PTS). One might want to say that we can get hyperintensionality for free, since it is a direct side product of its fundamental principles ('meaning via proofs'). In other words, nothing needs to be added, it is a basic built-in feature of many (if not every) PTS or PTS-related systems.

What do I mean by *proof-theoretic semantics* (PTS)? In this paper I will follow the rough but functional demarcation of PTS used by Jago (2014, p. 67) as well. Thus, we will take PTS as originating with the pioneering work of Gentzen (1935), and then followed up and further developed by Prawitz (1971; 1973), Dummett (1991), Schroeder-Heister (1991), Martin-Löf (1984) and others (see also Piecha and Schroeder-Heister 2016, Francez 2015, Read 2015).[1]

As the name itself suggests, the main idea of proof-theoretic semantics is that the central semantic notion—in terms of which meanings are given—is proof. In this regard, we can view PTS as an alternative to the more traditional truth-conditional/model-theoretic semantics that focusses on the notion of truth. More generally, we can view PTS as a part of the meaning-as-use paradigm (instigated mainly by Wittgenstein 1953, but also Sellars 1953) with close connections to inferentialism (Brandom 1994; 2000, Peregrin 2014) and inferential role semantics (Horwich 1995, Harman 1987, Field 1977, Block 1998).

For example, in PTS the meaning of logical conjunction '&' is not defined by the familiar truth table:

A	B	$A \mathbin{\&} B$
T	T	T
T	F	F
F	T	F
F	F	F

but through a collection of introduction and elimination rules (*I/E-rules*):

$$\frac{A \quad B}{A \mathbin{\&} B}\,\&I \qquad \frac{A \mathbin{\&} B}{A}\,\&E_1 \qquad \frac{A \mathbin{\&} B}{B}\,\&E_2$$

As can be easily checked, these rules capture the same behaviour of '&' as does the truth table above. For example: the first line of the table 'if A is true

[1] It is important to note that in its current state PTS is not really a unified framework with established methodology (see e.g., Piecha and Schroeder-Heister 2016 and Restall 2016), but rather an umbrella term that refers to a large family of variously related approaches. However, some important strides in this direction were recently undertaken e.g., by Francez (2015).

and B is true, then $A \& B$ is true' corresponds to the first rule $\&I$: 'if we can infer A and B, then we can also infer $A \& B$', etc.[2]

What do I mean by *hyperintensionality*? In this paper I will adhere to the basic specification provided by Cresswell (1975), the originator of the term itself:

> It is well known that it seems possible to have a situation in which there are two propositions p and q which are logically equivalent and yet are such that a person may believe the one but not the other. If we regard a proposition as a set of possible worlds then two logically equivalent propositions will be identical, and so if 'x believes that' is a genuine sentential functor, the situation described in the opening sentence could not arise. I call this the paradox of hyperintensional contexts. Hyperintensional contexts are simply contexts which do not respect logical equivalence. (Cresswell 1975, p. 25)

As mentioned in the quote above, a typical example of hyperintensional contexts are arguments involving so-called propositional attitudes (believe, know, desire, ...):

$$\frac{\text{Alice knows that } A \to A \text{ holds.} \qquad A \to A \Leftrightarrow \forall xyzn.((n > 2) \to \neg(x^n + y^n = z^n))}{\text{Alice knows that } \forall xyzn.((n > 2) \to \neg(x^n + y^n = z^n)) \text{ holds.}}$$

This inference is intuitively incorrect: even though $A \to A$ and Fermat's Last Theorem are logically equivalent, knowing one hardly entails knowing the other. On the standard truth-conditional (model-theoretic semantics, possible world semantics) approach, however, we would have to concede that this inference is actually warranted, since both $A \to A$ and Fermat's Last Theorem denote the same proposition, and thus have identical meanings.

I will say that a logical system is hyperintensional (or that it can deliver hyperintensional content) if it can distinguish between otherwise logically equivalent contents (= meanings). Therefore, the quest for hyperintensionality is the quest for finer-grained theory of meaning that uses a finer-toothed comb for individuation of meanings than logical equivalence.[3]

2 PTS and Hyperintensionality

2.1 Against Claim A

I this section I will try to show that:

[2] Definition of logical constants in PTS framework is not always as straightforward as this simple example might suggest. For more, see e.g., Došen (1980), Tennant (1978), Dummett (1991).

[3] For more on hyperintensionality, see also e.g., Duží et al. (2010) or Raclavský et al. (2015).

Counterclaim (**Ax**): PTS easily delivers hyperintensional contents.

And by 'easily' I mean without any significant technical hassle or prolonged philosophical argumentation.

Demonstration. Consider the following simple argument scheme, where A and B are two distinct but logically equivalent propositions and Δ is a proof that A:[4]

$$\frac{\begin{array}{c}\Delta \text{ is a proof that } A \\ A \Leftrightarrow B\end{array}}{\Delta \text{ is a proof that } B}$$

The argument above is, of course, generally incorrect. For example, consider theorems $A \to ((A \to B) \to B)$ and $A \to (B \to A)$. Even though they are logically equivalent, a proof of the former (see Π below) does not automatically count as a proof of the latter (see Ω below).

$$\Pi: \quad \cfrac{\cfrac{\cfrac{A \to B \qquad A}{B}\,{\to}E}{(A \to B) \to B}\,{\to}I}{A \to ((A \to B) \to B)}\,{\to}I \qquad\qquad \Omega: \quad \cfrac{\cfrac{A}{B \to A}\,{\to}I}{A \to (B \to A)}\,{\to}I$$

Thus the argument

$$\frac{\begin{array}{c}\Pi \text{ is a proof that } A \to ((A \to B) \to B) \\ A \to ((A \to B) \to B) \;\Leftrightarrow\; A \to (B \to A)\end{array}}{\Pi \text{ is a proof that } A \to (B \to A)}$$

is incorrect. Clearly, a proof starting with two premisses $A \to B$ and A and continuing with three steps carried out via consecutive application of rules $\to E$, $\to I$ and $\to I$ of (i.e., the proof Π) does not establish the theorem $A \to (B \to A)$.

So 'being a proof that', the fundamental notion of PTS, produces a context that does not respect logical equivalence. And since PTS identifies the meaning of a proposition with its proofs (more on this later), we are dealing with hyperintensional contents. Thus, I conclude that counterclaim (**Ax**) holds.

Remark. I take here the phrase 'Δ is a proof that A' to be effectively synonymous with other similar natural language reformulations such as 'Δ proves that A', 'that A is provable by Δ', etc. Although there might be some contexts where it is reasonable (or even desirable) to distinguish between these phrases, it has no practical impact on the general argument I am making here.

[4]It was probably Gödel (1938) who first worked with proofs in this explicit manner.

2.2 Against Claim B

I this section I will argue that:

> Counterclaim (**Bx**): In PTS it is not the case that every logically equivalent proposition has the same content.

Demonstration. Jago's argument for the original claim (**B**) goes as follows:

> The content of A consists of sets of representations, those from which 'A' can be inferred and those that can be inferred from A. [...]
>
> ...[S]uppose A and B are logically equivalent (they need not themselves be logical truths). Then any consequence of A is a consequence of B and vice versa, and for any set of premisses Γ of which A is a consequence, B is also a consequence (and vice versa). So again, A and B are assigned precisely the same content, on this approach. [...]
>
> I do not mean to dismiss proof-theoretic semantics, or conceptual-role semantics more generally. My point is that it does not easily deliver hyperintensional contents. (Jago 2014, pp. 66–68)

Let us try to make it even more precise. I will denote the (proof-theoretic) meaning of A as $[\![A]\!]$,[5] derivability relation as $\vdash$ and call sets of representations from which A can be inferred as *premisses* of A (denoted as $pre([\![A]\!])$; $pre([\![A]\!]) =_{\text{def.}} \{\Gamma \mid \Gamma \vdash A\}$) and sets of representations that can be inferred from A as *conclusions* of A (denoted as $con([\![A]\!])$; $con([\![A]\!]) =_{\text{def.}} \{C \mid A \vdash C\}$). Thus, we get:

1. Assumption: $[\![A]\!] =_{\text{def.}} pre([\![A]\!]) \cup con([\![A]\!])$
2. Suppose: $A \Leftrightarrow B$
3. Then: $A \vdash C \Leftrightarrow B \vdash C$
4. Also: $\Gamma \vdash A \Leftrightarrow \Gamma \vdash B$
5. Therefore: $[\![A]\!] = [\![B]\!]$

The main issue I have with this argument is that to my knowledge there is no PTS system that actually upholds the first assumption (1.) upon which the whole argument stands. In other words, the definition of PTS meaning provided by Jago is misleading.

[5] I borrow this notation and the notion of 'reified' proof-theoretic meaning from Francez (2014*b*; 2015).

In PTS environment concerned with non-empirical discourse the central semantic role is always played by proofs.[6] But not just any proofs, what is required are canonical proofs. And it is these canonical proofs (or more generally, derivations, arguments) that are regarded as establishing meaning of propositions. For example, to quote Dummett:

> A canonical proof is a proof of the specially restricted kind in terms of which the meanings of mathematical statements, and of the logical constants in particular, are given... (Dummett 1991, p. 177)

Or more recently, Francez:

> The meaning of a sentence [...] is based on its [I-]canonical derivations from its grounds for warranted assertion. (Francez 2015, p. 41)

So, e.g., for Francez $[\![A]\!]$ does not denote a set of premisses and conclusions of A but a set of its canonical derivations (see Francez 2015, p. 38, p. 183).[7] On this approach it is easy to check that the meanings of logically equivalent propositions are not really the same.

For example, $A \,\&\, B$ is logically equivalent with $B \,\&\, A$ (hence premiss (2.) is satisfied), and same things can be derived from them (i.e., A and B, thus, premiss (3.) is satisfied), both can be derived from same premisses (from A and B, see derivations Σ and Φ below, thus, premiss (4.) is satisfied), however, that does not entail that they have the same meaning – in fact, their meanings are different, because their respective proofs are different!

$$\Sigma\!: \ \dfrac{A \qquad B}{A \,\&\, B}\,\&I \qquad\qquad \Phi\!: \ \dfrac{\dfrac{\dfrac{A \qquad B}{A \,\&\, B}\,\&I}{B}\,\&E_2 \qquad \dfrac{\dfrac{A \qquad B}{A \,\&\, B}\,\&I}{A}\,\&E_1}{B \,\&\, A}\,\&I$$

Therefore, we can have both $pre([\![A \,\&\, B]\!]) = pre([\![B \,\&\, A]\!])$ and $con([\![A \,\&\, B]\!]) = con([\![B \,\&\, A]\!])$ but simultaneously $[\![A \,\&\, B]\!] \neq [\![B \,\&\, A]\!]$. In other words, even though their premisses and conclusions are the same, their meanings are not

[6] See e.g., Dummett (1991), Prawitz (2006; 1973; 1977), Schroeder-Heister (1991), Francez (2015), Read (2015), Martin-Löf (1984), Sundholm (2000). Although, recently there were also some developments, see e.g., Prawitz (2012) or Usberti (2006). This approach can be extended towards empirical discourse as well, see e.g., Więckowski (2016). But, as Dummett pointed out, it can sometimes lead to an 'unattractive messiness' (Dummett 1991, p. 279).

[7] Whether in a normal form or not – a proof is said to be in a normal form if all unnecessary 'detours' have been removed, i.e., all immediate applications of the corresponding I and E rules have been taken out. Otherwise, the proof is in a non-normal form. This process is usually referred to as normalization or detour conversion. See e.g., Negri and von Plato (2001, p. 9).

due to the fact that they were derived differently as can be witnessed by their distinct derivation trees.[8] Hence, I conclude that counterclaim (**Bx**) holds.

Remark. I assume here the PTS 'industry standard', i.e., Prawitz's (1965) natural deduction as the logical system in the background, where derivation trees are sequences, i.e., ordered lists $\langle A_1, A_2, \ldots, A_n, B\rangle$ written as $\dfrac{A_1\ A_2\ \ldots A_n}{B}$ where $A_1, A_2, \ldots, A_n$ are premisses and B their conclusion. For more, see Prawitz (1965, p. 22), especially the footnotes. Also note that once we utilize proofs as vehicles for meaning, the topic of identity of proofs rises to prominence – it effectively becomes the topic of sameness of meaning. Identity of proofs is far from a simple issue, however, since it has no immediate effect on the argument that PTS can deliver hyperintesional contents (no matter what particular criteria of proof synonymy we adopt), we satisfy ourselves here with this basic approach, i.e., same derivation tree modulo normalization process = same proof. There are, of course, ways how we can augment this approach further. For example, we could utilize so-called Curry-Howard isomorphism (CHi), more specifically, natural deduction decorated with λ-terms serving as proof-objects ('reified proofs'). This would allow us to employ finer benchmarks for identity of proofs than detour convertibility, which—with CHi—corresponds to β-reduction (e.g., $\lambda x.f(x)a$ and $f(a)$ are regarded as an equivalent proof-objects). For example, we could state that identical proofs are those that are α-equivalent, thus e.g., the proof-objects $\lambda x.x$ and $\lambda y.y$ will be considered identical. Or maybe we choose η-equivalence as the criterion of synonymy, therefore $\lambda x.f(x)$ and f will be viewed as the same proof, etc. These λ-conversions then essentially become a potentiometer for fine-tuning the proof synonymy threshold. Alternatively, we could rely on the so-called grounds as does e.g., Francez (2015). For more on this topic, see also e.g., Došen (2003), Girard (1989), Martin-Löf (1975), Prawitz (1971), or Straßburger (2006).

One final note, as I mentioned at the beginning of the paper, Jago subsumes PTS under CRS. So, as one reviewer correctly pointed out, even though Jago is incorrect about PTS, he still might be right about some other accounts he subsumes under CRS. This is true, of course, although as far as I can tell it is none of those Jago mentions: Field (1977), Harman (1987), Miller and Johnson-Laird (1976) all seem to account for hyperintensionality in some manner. For example:

[8] By 'distinct' I mean that i) Σ and Φ are syntactically different and ii) Σ is not normalizable into Φ or vice versa. This condition ensures that normalization remains a meaning preserving procedure. For example, as suggested by an anonymous reviewer, consider a new proof Ψ which is essentially Φ but with the sub-derivations and the conjuncts of the conclusion swapped ($\&E_1$ on the left, $\&E_2$ on the right). Then Ψ can be normalized into Σ. Hence, even though they are syntactically different derivation trees, they are 'semantically' equivalent proofs since they can be transformed into each other.

> One obvious way to do this [make the account of conceptual role more fine-grained] is to say that two sentences have the same *fine-grained conceptual role* if (i) they have the same conceptual role [...], and (ii) they are built up in the same way from parts with the same conceptual roles. (Field 1977, p. 397)

> If the issue is whether there can be two different but logically equivalent beliefs, the answer is obviously 'Yes'. (Harman 1987)

And although I was not able to find a direct textual evidence from Miller and Johnson-Laird (1976)—given Johnson-Laird's involvement with procedural semantics (Johnson-Laird 1977)—distinguishing between logically but not procedurally equivalent contents should come quite naturally to them as well.

3 Conclusion

In this paper I tried to show that contrary to Jago's claims hyperintensionality is something that comes quite naturally to PTS. This is due to the fact that (canonical) proofs are not only used as primary semantic values of propositions, but also as a means of their individuation. Shortly put, proofs can be used to distinguish between logically equivalent propositions. For example, even though Fermat's Last Theorem is logically equivalent to $A \to A$ (or any other theorem for that matter), we can tell them apart by looking at their proofs.

There are already number of specific approaches with more refined accounts of proofs (both PTS and PTS-related), which take this fact explicitly into account (either under the heading of hyperintensionality, or simply intensionality, semantic finegrainedness, granularity of meaning, etc.[9]). Consequently, each of these approaches formalizes our pre-theoretic 'Δ is a proof that A' in different ways. For example, for Martin-Löf it becomes a judgement $\Delta \in A$, where Δ is a proof object and A its type, Artemov develops it into a justification assertion $\Delta : A$, where Δ is a justification term and A a formula and Francez essentially replaces it with a more specific 'Δ is a set of canonical proofs of A'. I will not demonstrate the hyperintensional capabilities of these systems here, but analogous results to those presented above can be obtained in them as well.

Acknowledgements

I would like to thank the anonymous reviewers for their helpful remarks and suggestions.

[9]See e.g., Francez's (2015, 2014*a*) natural deduction proof systems, Martin-Löf's (1984) constructive type theory (see also Więckowski 2015, Luo 2014), Gabbay's (1996) labelled deductive systems, or Artemov's (2005) justification logic.

The work on this paper was supported by the grant of the Czech Science Foundation (GA ČR) "Semantic Notions, Paradoxes and Hyperintensional Logic Based on Modern Theory of Types," registration no. GA16-19395S.

References

Artemov, S. and Nogina, E. (2005), 'Introducing justification into epistemic logic', *Journal of Logic and Computation* **15**(6), 1059–1073.

Block, N. (1998), Conceptual role semantics, *in* E. Craig, ed., 'Routledge Encyclopedia of Philosophy', Routledge, pp. 242–256.

Brandom, R. (1994), *Making it Explicit: Reasoning, Representing, and Discursive Commitment*, Harvard University Press.

Brandom, R. (2000), *Articulating Reasons*, Harvard University Press.

Cresswell, M. J. (1975), 'Hyperintensional logic', *Studia Logica* **34**(1), 25–38.

Došen, K. (1980), Logical constants: an essay in proof theory, PhD thesis, University of Oxford.

Došen, K. (2003), 'Identity of proofs based on normalization and generality', *The Bulletin of Symbolic Logic* **9**(4), 477–503.

Dummett, M. (1991), *The Logical Basis of Metaphysics*, Duckworth, London.

Duží, M., Jespersen, B. and Materna, P. (2010), *Procedural Semantics for Hyperintensional Logic: Foundations and Applications of Transparent Intensional Logic*, Logic, Epistemology, and the Unity of Science, Springer.

Field, H. (1977), 'Logic, meaning, and conceptual role', *Journal of Philosophy* **74**(July), 379–409.

Francez, N. (2014*a*), The granularity of meaning in proof-theoretic semantics, *in* N. Asher and S. Soloviev, eds, 'Logical Aspects of Computational Linguistics: 8th International Conference, LACL 2014, Toulouse, France, June 18-20, 2014. Proceedings', Springer Berlin Heidelberg, Berlin, Heidelberg, pp. 96–106.

Francez, N. (2014*b*), 'Views of proof-theoretic semantics: reified proof-theoretic meanings', *Journal of Logic and Computation* .

Francez, N. (2015), *Proof-theoretic Semantics*, College Publications.

Gabbay, D. M. (1996), *Labelled Deductive Systems*, number v. 1 *in* 'Oxford Logic Guides', Oxford University Press, USA.

Gentzen, G. (1935), 'Untersuchungen über das logische Schließen. I', *Mathematische Zeitschrift* **39**(1), 176–210.

Girard, J.-Y. (1989), *Proofs and Types*, Cambridge Tracts in Theoretical Computer Science 7, Cambridge University Press.

Gödel, K. (1938), *Unpublished Essays and Lectures (Kurt GÃűdel Collected Works: Volume III)*, Oxford University Press, chapter Vortrag bei Zilsel/Lecture at Zilsel's, pp. 86–113.

Harman, G. (1987), (Nonsolipsistic) Conceptual Role Semantics, *in* E. Lepore, ed., 'New Directions in Semantics', London Academic Press, pp. 55–81.

Horwich, P. (1995), 'Meaning, use and truth: On whether a use-theory of meaning is precluded by the requirement that whatever constitutes the meaning of a predicate be capable of determining the set of things of which the predicate is true and to which it ought to be applied', *Mind* **104**(414), 355–368.

Jago, M. (2014), *The Impossible: An Essay on Hyperintensionality*, Oxford University Press UK.

Johnson-Laird, P. N. (1977), 'Procedural semantics', *Cognition* **5**(3), 189–214.

Luo, Z. (2014), Formal semantics in modern type theories: Is it model-theoretic, proof-theoretic, or both?, *in* N. Asher and S. Soloviev, eds, 'Logical Aspects of Computational Linguistics: 8th International Conference, LACL 2014, Toulouse, France, June 18-20, 2014. Proceedings', Springer Berlin Heidelberg, Berlin, Heidelberg, pp. 177–188.

Martin-Löf, P. (1975), About models for intuitionistic type theories and the notion of definitional equality, North-Holland Publishing Company, pp. 81–109.

Martin-Löf, P. (1984), *Intuitionistic type theory*, Studies in proof theory, Bibliopolis.

Miller, G. A. and Johnson-Laird, P. N. (1976), *Language and Perception*, Harvard University Press.

Negri, S. and von Plato, J. (2001), *Structural Proof Theory*, Cambridge University Press.

Peregrin, J. (2014), *Inferentialism: Why Rules Matter*, Palgrave Macmillan.

Piecha, T. and Schroeder-Heister, P. (2016), *Advances in Proof-Theoretic Semantics*, Springer International Publishing.

Prawitz, D. (1965), *Natural Deduction: A Proof-theoretical Study*, Dover Books on Mathematics Series, Dover Publications, Incorporated.

Prawitz, D. (1971), Ideas and results in proof theory, North-Holland Publishing Company, pp. 235–309.

Prawitz, D. (1973), Towards a foundation of a general proof theory, Studies in logic and the foundations of mathematics, North-Holland Publishing Company, pp. 225–250.

Prawitz, D. (1977), 'Meaning and proofs: On the conflict between classical and intuitionistic logic', *Theoria* **43**(1), 2–40.

Prawitz, D. (2006), 'Meaning approached via proofs', *Synthese* **148**(3), 507–524.

Prawitz, D. (2012), 'The epistemic significance of valid inference', *Synthese* **187**(3), 887–898.

Raclavský, J., Kuchyňka, P. and Pezlar, I. (2015), *Transparent Intensional Logic as Characteristica Universalis and Calculus Ratiocinator [in Czech]*, Brno: Masaryk University (Munipress), Brno.

Read, S. (2015), Proof-theoretic validity, *in* C. R. Caret and O. T. Hjortland, eds, 'Foundations of Logical Consequence', Oxford University Press, pp. 136–158.

Restall, G. (2016), 'Thomas Piecha and Peter Schroeder-Heister (eds.), Advances in Proof-Theoretic Semantics [review]', *Notre Dame Philosophical Reviews: An Electronic Journal* **2016.05.15**.

Schroeder-Heister, P. (1991), 'Uniform proof-theoretic semantics for logical constants', *Journal of Symbolic Logic* **56**, 1142.

Sellars, W. (1953), 'Inference and meaning', *Mind* **62**(247), 313–338.

Straßburger, L. (2006), Proof nets and the identity of proofs, Technical report, Institut National de Recherche en Informatique et en Automatique.

Sundholm, G. (2000), 'Proofs as acts versus proofs as objects: Some questions for Dag Prawitz', *Theoria* **64**(2-3), 187–216.

Tennant, N. (1978), *Natural Logic*, Edinburgh: Edinburgh University Press.

Usberti, G. (2006), 'Towards a semantics based on the notion of justification', *Synthese* **148**(3), 675–699.

Więckowski, B. (2016), 'Subatomic natural deduction for a naturalistic first-order language with non-primitive identity', *Journal of Logic, Language and Information* **25**(2), 215–268.
URL: *http://dx.doi.org/10.1007/s10849-016-9238-7*

Więckowski, B. (2015), 'Constructive belief reports', *Synthese* **192**(3), 603–633.

Wittgenstein, L. (1953), *Philosophical investigations*, Macmillan.

IVO PEZLAR
Department of Philosophy
Faculty of Arts
Masaryk University
Arne Nováka 1
602 00, Brno, Czech Republic
pezlar@phil.muni.cz
http://ivo.pezlar.com